R. GIRWIDZ, M. VOGEL, C. SPANNAGEL, & J. ENGEL

COMPREHENSION OF GRAPHS – SUPPORTED BY SUPPLANTATION OF POINT-TO-OBJECT OPERATIONS

Abstract. Younger students often fail to comprehend and process diagrams. They have difficulties to integrate the abstract representation and the modeled problem in an appropriate way. We propose to help students by offering them an adequate external representation using supplantation. The principle is to simulate mental operations externally. Based on an information processing model we show how supplantation can be used to help students to work with diagrams. In this process interactivity plays an important role. We identify two types of mental operations on graphs to be supplanted. Subsequently, we discuss the results of a pilot study to utilize empirical results about conditions and advantages of applying supplantation. First results indicate that supplantation is helpful especially for not so gifted students.

1. MOTIVATION

Mathematics and science teachers know the difficulties students have in secondary level schools, when working with functions and functional dependencies. Understanding abstract mathematical representations of x-y-relations is very hard for novices. There are several skills required to solve tasks with representations like tables, graphs, and diagrams going beyond an elementary level, as described by Wainer (1992): Students have to think about the given problem, and in case of being inexperienced in working with tables, graphs and diagrams, they have to think about this representation, too. In other words – they have to cope with the problem and its representational code in addition, whereby the latter is a prerequisite of the former. That means, if they fail in understanding the code in which the problem is presented, they have no chance to solve the problem itself. Without specific support, weaker students will fail to go beyond a very low level of working with diagrams, for example simply identifying associated x- and y-values.

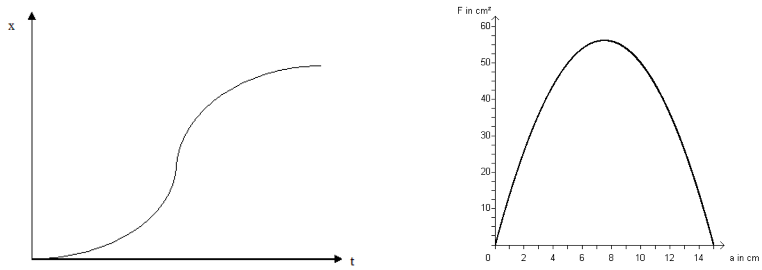

Figure 1a + b. Two examples of graphs some students fail to interpret properly

Two striking examples of our own teaching experiences illustrate the problem: students were confronted with the diagram shown in Figure 1a and the additional explanation that the slope of the curve represents the speed of a cyclist. Then they were asked to explain the motion in a qualitative way. Some students did this properly by describing *acceleration,* or *deceleration* respectively, for several points of the graph. But some students argued that the cyclist first turns to the left and then turns to the right. Apparently these students failed to interpret the graph, assuming they would look at the cyclist from above. Without help they had no chance to solve further problems like finding the point of maximum or minimum speed. In another example students were asked to model an optimization process given by the following description: *Toby wants to build a rectangular cage for his rabbit. Material for a fence, 30 meters long, is available. How should he build the cage to fence in the largest area possible?* A graph was given to show how the area depends on the length of one side (Fig. 1b). When we asked the students about their interpretations, it became evident that many difficulties arise because they misunderstand the graph. There were even some extremes, seeing the curve of the jumping rabbit or seeing the fence from a top view.

In the examples described above, some students did not recognize the abstract character of this type of representation. Those students are not able to apply the necessary mental transformations to extract the encoded information. They interprete the abstract visualization in a non-abstract manner. Thus, several questions for practical work arise:

- How can we bridge the gap between the concrete problem and its abstract representation?
- How can the mental operations and transformations be activated that are necessary for the work with logical pictures?
- In which way can the problem be presented to help students building a link between diagrams and the objects of the problem domain *without simplifying the mathematical problem too much*?

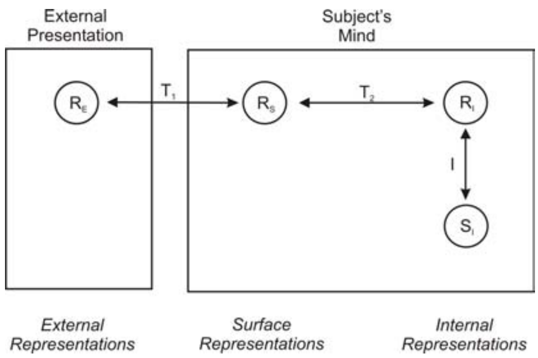

Figure 2. A model of information processing, including the processes of transformation (processes T_1 and T_2) and the process of integration / interaction (process I)

In the discussion how to support the development of cognitive skills, often interactive learning material is suggested. Interactive presentations give students the opportunity to create and validate their own concepts and models by changing the kind of representation on demand. In this article we focus on interactive presentations and a major question in the following sections is:

- How can interactive learning material support understanding of graphs?

2. THEORETICAL BACKGROUND

2.1. Model of Information Processing

To support students' learning, it is important to understand how they perceive and interpret visual information. Several information processing models describe the transformation from external to internal representations (Mayer, 1997; Mayer, 2001; Schnotz & Bannert, 1999). A first step is the development of a surface representation based on the presented material (text base, image base). A central proficiency for this is the selection of relevant words and images (Mayer, 1997). Schnotz and Bannert discuss semantic processing and thematic selection (Schnotz & Bannert, 1999). Further on, text and image bases are transformed to internal verbally-based and visually-based models (Mayer, 1997). Organizing information, parsing of symbols, mapping of analogue structures, mental model construction and inspection are important parts of this process (Schnotz & Bannert, 1999). But internal representations are not just copies of the external representations. They are *constructed* actively by the individual (Mayer, 1997; Schnotz & Bannert, 1999). Schnotz and Bannert emphasize that this is done with regard to possible future demands. If there are more internal representations available at the same time, they are related to each other. Mayer uses the term "integration", Schnotz and Bannert speak of mutual supplementation via the construction of mental models and model inspection (Schnotz & Bannert, 1999).

A schematic visualization of these processes is shown in Figure 2. At first, the external representation R_E is transformed into a surface representation R_S, which is then translated to an internal representations R_I. If a second internal representation S_I is available at the same time, the two representations are integrated. When students have to solve a problem using graphs, they might have difficulties at two points of the information processing model described above: They might fail in transforming the external into an internal representation of the graph (processes T_1 and T_2 in Figure 2), or they can fail in integrating the internal representation of the graph with the internal representation of the concrete problem (process I in Figure 2).

2.2. Supplantation

Going on from the introductory examples and the information processing model we now propose an instructional design which is intended to avoid the problems mentioned. If students are not able to perform a mental operation, like linking an abstract representation with the concrete problem, this operation can be visualized externally by media. The external modeling can help students to construct adequate internal models of the operation. This is called supplantation, and stands for the external representation of operations which cannot be performed internally by the individual. Research on supplantation has been done by Salomon (1994).

> "When the model overtly supplants the internal process, it can be learned by observation or […] be internalized." (Salomon, 1994, p. 232)

And further:

> "When a code supplants mental operations that learners cannot yet perform well on their own, we might expect them to learn the code." (Salomon, 1994, p. 134).

From this point of view it should be emphasized that, in addition to the information about the objects themselves, the externally represented link between these objects becomes important.

Designing a supplantation-scenario has to take into account the previous knowledge of the learners. If the learner does not know the represented objects, his attention will be drawn to these objects first, and not to the processes that should be supplanted. If the represented objects are not well known, they have to be introduced before (Salomon, 1994). Additional factors determining the efficiency of the method are to be mentioned: the intended connections and transformations should be recognizable without greater effort, and students should expect that the offered coding is useful for them (Salomon, 1994). Ausburn & Ausburn (1978) suggest a three-step process to develop an instructional design to consider *which process*, for *whom* and *how* supplantation has to be done.

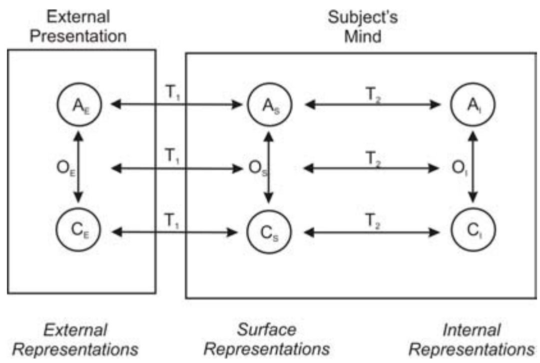

Figure 3. The supplantation of the operation O_I in the external representation

Showing the connections in the external representation, the integration itself becomes a process which has to be transformed into an internal representation. This is shown in Figure 3. At first the external operation (O_E) will be translated into a surface representation (O_S) which will then be transformed into an internal representation of the operation (O_I). The simultaneous presentation of an external graph (A_E) and an external representation of the concrete problem (C_E) also reduce the risk that the learner takes one for the other and reads the abstract representation in a non-abstract way.

2.3. Multiple External Representations

When instructional designers present an abstract graph, the concrete object and the integration of them, they use multiple external representations. Ainsworth (1999) categorizes multiple external representations (MERs) based on their functions: using MERs to support complementary processes, to constrain interpretations, and to construct deeper understanding. In the context described above, a subcategory of the second category is relevant: using MERs so that a familiar representation constrains interpretation of a second unfamiliar representation.

> "One rationale for exploiting a familiar representation is to support the interpretation of a less familiar or more abstract one and to provide support for a learner as they extend, or revise misconceptions in, their understanding of the unfamiliar." (Ainsworth, 1999, p. 139)

It should be sure that learners can grasp the relation between the representations (Ainsworth, 1999). This can be done with *dynamic linking* of the representations in a computer program, which means that if the learner changes details in one representation, the corresponding changes will appear in the second representation simultaneously. This is an example for using supplantation as described above. With growing experience of the learner, such a support should be faded out. The level of scaffolding should depend on the student's performance (Ainsworth, 1999). The success of this instructional technique can be assessed by determining the ability of students to use the former unfamiliar representations afterwards.

Kozma's research on multiple representations leads to some design principles for the use of multiple representations. He discusses the possibility to support connections across representations and connecting different representations to the physical phenomena they stand for.

> "Linkages can be accomplished by any of a variety of symbolic conventions that would allow students to map surface features of one representation onto those of another. For example, the actions that a student takes with one representation can correspond to certain outcomes in another representation." (Kozma, 2003, p. 219).

A good agreement with the aim of dynamic linking can be stated.

3. SUPPLANTATION OF POINT-TO-OBJECT OPERATIONS

As described above, supplantation can bridge the gap between the concrete problem and its abstract representation. To understand in which way a point of the graph is related to a situation or object in the concrete problem domain may be crucial for graph interpretation. There are at least the following two operations where supplantation may be beneficial:

- Imagination of the concrete object or situation related to a certain point of the graph
- Imagination how the concrete object or situation changes when a point of the graph is moved to the left or the right

We call these operations *point-to-object operations* on graphs. The first category is a *static point-to-object operation*, and the second one a *dynamic point-to-object operation*. The supplantation of point-to-object operations can be realized by computer programs which perform these operations externally using dynamic linking. Figure 4 gives an example:

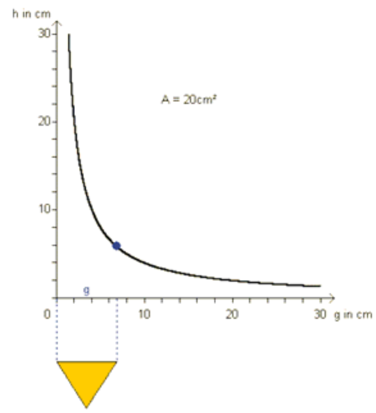

Figure 4. Interactive computer presentation of the triangle problem using supplantation

The graph shows the relation between the length of the baseline g and the height h of a triangle with constant area. Beneath the x-axis, the triangle is shown that corresponds to the marked point on the graph. The point can be moved along the graph with the computer mouse. Simultaneously, the triangle changes. This way both point-to-object operations described above can be performed externally.

4. A PILOT STUDY

4.1. Hypotheses

Based on the theoretical considerations described above the following pilot study has been designed. The intention was to investigate the impact of offering supplantation to students when interpreting graphs. Our first hypothesis is:

> *Students who work with presentations using the supplantation principle (V1) do better on tasks requiring them to interpret graphs (task1) than students who use graph-only presentations (V2):*

Denoting the respective probabilities by p (V1_task1) and p (V2_task1), we can formulate hypothesis 1 as:

$$H1: \quad p(V1\_task1) \quad > \quad p(V2\_task1)$$

This first hypothesis asks for performance tests of students who are confronted with graphs after they had worked with supplantation examples.
One may assume that these students try to transfer the observed processing principles presented with supplantation technique. Our second hypothesis states:

> *Students who have been supported with supplantation in task1 are more successful in performing a similar task (task 2) on a graph-only presentation than students who have not been supported before:*

$$H2: \quad p(V1\_task2) \quad > \quad p(V2\_task2)$$

Besides the different representations offered in the pilot study, the results are expected to be influenced by the general mathematical competencies of the students.

4.2. Method

4.2.1 Subjects and Procedure
44 middle-school students (9th grade) from two classes with different mathematics teachers participated in the pilot study. The students were randomly assigned to one of two groups. The first group V1 consisted of 23 students who worked in task 1 on four items with supplantation. 21 students of the second group V2 worked on the same item without supplantation. Afterwards task 2, consisting of six items, was administered to both groups without any support. Before assigning the tasks to the students they were instructed about the procedure. Time was not limited. The study took place during the regular lessons.

4.2.2. Material
The participants were given two graphical problems. The first (task1) was about the relation between the height and the base of a triangle with a constant area (see Figure 4). The task was presented on laptops. The presentation of the problem with supplantation allowed the students to work with it interactively, performing *point-to-object operations* externally. The students of group V2 worked with a graph-only representation. The second problem (task2) was the "cage-problem" as described in section 1. This problem was given to the students of both groups V1 and V2 in the "traditional" way, only using paper, pencil, and a sheet of paper showing a graph relating one side of the rectangle to the encased area. In both passes the students got a multiple-choice questionnaire about information directly coded in the graph and questions about the represented object.

4.2.3. Data
The main variable of interest was the number of correct answers for the items of task 1 and task 2 (or the respective percentages) for all 44 students. Group membership (V1 or V2), gender (*x males, y females*), age (*average age*) and grade in

mathematics were recorded as covariables. Furthermore, the time required to finish the tasks, was recorded (*M=12.73 minutes, SD= 2.03 minutes*). Students' actions on the laptops were recorded by the capture & replay tool "*CleverPHL*", a computer program which records the user behavior and allows to replay it later (Spannagel, 2003).

4.3. Results

In the first pass (task 1) the students who worked with supplantation reached higher scores (M=2.13, SD=1.39) than the other ones (M=0.76, SD=0.89, Fig. 5). The two-sided Wilcoxon-rank-sum-test identifies this difference as significant (p=0.012). This result agrees with our hypothesis H1.

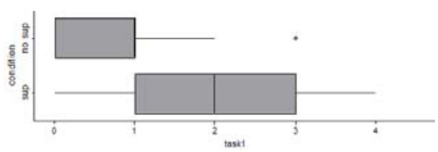

Figure 5. Boxplots comparing supplantation with graph-only group (task 1)

In the second task the students who worked with supplantation before had slightly higher scores (M=1.96, SD=1.55) than those students who worked with graph-only presentations in both passes (M=1.62, SD=1.20, Fig. 6). However, because of the high variances the difference is not significant (p=0.5464).

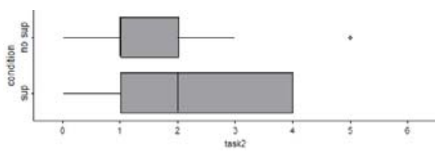

Figure 6. Boxplots comparing supplantation with graph-only group (task 2)

It is interesting to look at correlations between performances on the two tasks. While Spearman's rank correlation coefficient between task 1 and task 2 for the group V1 is remarkably low (r=0.0461) and not significantly different from zero, correlation for V2 amounts to 0.1530, an indication for the coherence of the two tasks. Our interpretation of the low correlation coefficient for V1 is that not all students may benefit equally from supplantation.

To investigate this hypothesis more detailed, we looked at the results taking into account mathematical competencies as covariable. Dividing the students into a group "+" with high abilities in mathematics (German grade 1, 2 or 3 in the last transcript) and a group "-" of low performance (German grade 4, 5 or 6), the Wilcoxon-rank-sum test shows no significant difference (p=0.2933) for task1 between the high and low performers, indicating that the immediately available supplantation is a helpful tool for poorer mathematics students (Fig. 7). However, in task 2 students with high mathematical skills outperformed the less mathematically skilful almost significantly (Wilcoxon-rank-sum-test p=0.0742).

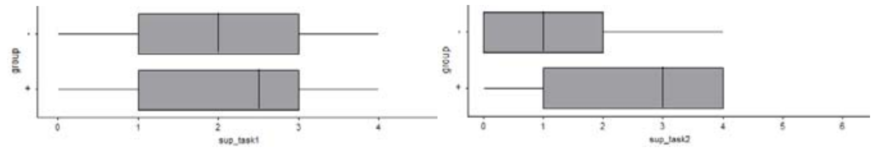

Figure 7: Boxplots comparing achievement on task 1 and task 2 of students with good math grades (+) and those with poor grades (-)

There are two possible explanations for the observed discrepancy: the mathematically skillful outperform the poor mathematicians in task 2 just because they are the better mathematicians or because they succeed in transferring the supplanted principles.

To investigate possible explanations more detailed, we fitted a linear model to the data

$$task\_i = math\_grade + condition, \; i = 1,2,$$

where *condition* is a variable indicating the support through supplantation.
Computation leads to the following regression coefficients:

Table 1. Regression coefficients

	Intercept	*Math grade*	*Condition*
Task 1	3.0079	-0.3025	-1.2809
Task 2	3.9912	-0.6115	-0.3025

Notable is the fact, that for task 1 only the coefficient of the variable condition (supplantation: yes or no in task 1) is significantly different from zero (p=0.0007) while for task 2 only the coefficient of the variable Math_grade differs significantly from zero (p=0.0124). We interpret these findings as an indication that direct supplantation is an valuable technique to support especially the mathematically weaker students. Whereas in task 2 general mathematical competencies were more decisive, and it was of secondary importance whether students had worked with the supplantation program in task 1 or not.

4.4. Discussion

The result of task 1 supports hypothesis H1. Working with supplantation of point-to-object-operations enables students to perform better than those students who only work with a usual graph representation. The records of the user behavior supply evidence that students frequently used the possibility of interactivity.

In the second task there is no significant difference due to supplantation and no evidence to maintain hypothesis H2. Without direct supplantation of point-to-object operations the students were obliged to perform these operations by themselves. The results of the second task indicate that a single interaction with a program of point-to-object operations is insufficient for transfer. One explanation could be that, according to requirements, students either used the abstract or the concrete representation. Thus they might not have realized their coherence. The low correlation coefficient between task 1 and task 2 for group V1 could indicate a non-sufficient influence of supplantation. However, these are conjectures for possible explanations and the current study doesn't provide sufficient empirical evidence.

With regard to the mathematical competencies, it is remarkable that the difference in performance between the subgroup "+" (better mathematicians) and the subgroup "-" (not so good mathematicians) is small in the first task for V1 (the population that worked with supplantation). Whereas in the second task there is a difference nearly reaching significance. This may indicate that students with lower mathematical competencies especially benefit from direct supplantation of point-to-object operations: they coped with the first task as well as the students of the group "+" did. But the results also indicate that without further help students do not transfer their experiences with directly presented point-to-object operations to situations where only a graph without assistance is given.

5. CONCLUSIONS AND FUTURE WORK

In this article we proposed the supplantation of point-to-object operations to support the comprehension of graphs. The instructional design has been developed based on a model of information processing, on research on supplantation and on research on learning with multiple external representations. Our pilot study is a first approach to show effects on learning with supplantation of point-to-object operations. This study was mainly designed to gain basic information about the effectiveness of this support. Regarding to the results and to the discussion above we can summarize:

- The supplantation of point-to-object operations has the potential to advance the comprehension of graphs.
- For an internalization of externally represented point-to-object operations one single presentation is not sufficient. It needs a broader base.

This leads to questions for further studies:

- Do students who have worked with *more than one supplantation example* transfer the point-to-object operations in situations with graph-only presentations more easily?
- Does *explicit instruction* on point-to-object operations help to perform the transfer?

In addition, the following open questions arise:

- What are the effects of supplantation on the cognitive load for experts in comparison with beginners (see Chandler & Sweller 1991)?

- Do interactive presentations outmatch simple animations which only show the processing without interactivity?
- Are there other mental operations on graphs than the described point-to-object operations that can be assisted using the supplantation principle?

The investigation of these and further questions is an objective of the research group “Interdisciplinary Learning with Digital Media” at the University of Education in Ludwigsburg. Supported by the state of Baden-Württemberg this research group develops, implements, and evaluates multimedia presentations for interdisciplinary classes in secondary schools.

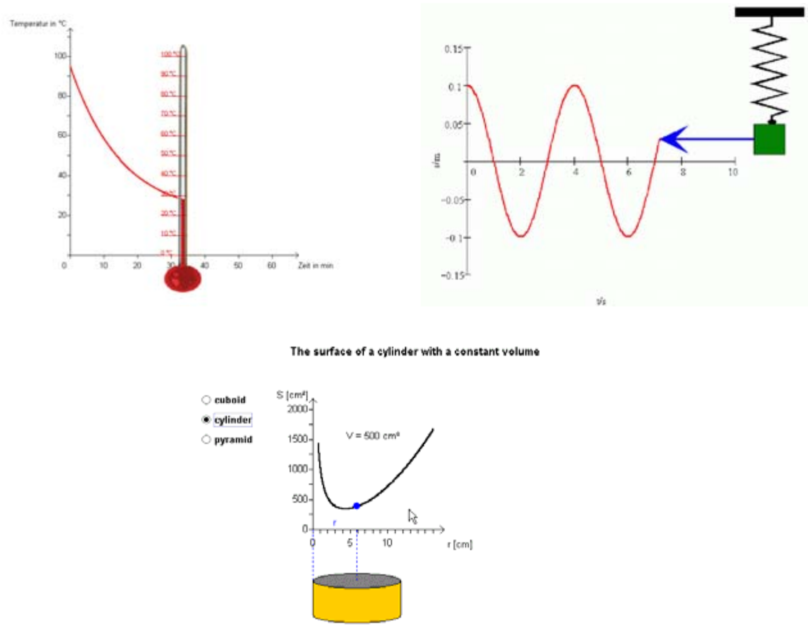

Figure 8: Supplantation of point-to-object operations in interdisciplinary and mathematical contexts

Interdisciplinary contexts are appropriate for instructional material which supplants point-to-object operations on graphs, because mathematical representations like diagrams can be dynamically linked with objects of physics or biology. Some examples are shown in Figure 8: The first illustration shows how temperature decreases with time when a body is cooling down. The picture on the right illustrates the s(t)-diagram of an oscillator. The picture at the bottom demonstrates how the surface of a cylinder with constant volume depends on its radius, a topic which is discussed in our interdisciplinary learning environment.

AFFILIATIONS

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