

The Role of Interactive Visualisations in the Development of Concepts of Logic

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Abstract: The main objective of the experiments described in this paper is to examine whether instructional variables often used in learning empirical sciences can also be used to facilitate the development of knowledge and skills in the formal sciences, particularly in learning logic. In learning empirical sciences, many positive effects have been reported which can be accounted for by the use of visualisation techniques, multiple representations and possibilities for the learners to interact with representations. It is our view, however, that formal sciences which are often described in a purely symbolic way, can also profit from these new techniques. This paper describes the effect of visualisations and interaction with these visualisations in learning logic. The results of the experiments described show that these instructional variables indeed lead to better learning effects.

Introduction

As computer technology has advanced during the past decades, the use of the computer as a medium for instructional communication has increased resulting in a renewed interest in the design of instructions and instructional communications. The computer can provide all kinds of representations of objects and interactions with these objects. As a consequence, the effect of different types of instructional variables on the development of knowledge and skills has been studied and special interests for the effects of visualisation techniques, multiple representations and possibilities for the learners to interact with representations, have emerged. Researchers started to focus on theories that address issues such as authentic problem solving and visualising problem situations. However, studies using these new ideas and theories often use materials from the empirical sciences such as biology, chemistry and physics, in which representations are made on both a pictorial and a symbolic level. The concepts and operations in the formal sciences, such as mathematics and logic, are often described in a purely symbolic way. As, for many learners, the instructions with only symbols as representations lead to a lack of understanding the formal rules and operations (e.g., Pintrich, 1990; White, 1993), the instructional communication for formal sciences is studied in the light of the new ideas and theories. More precisely, the effect of visualisations and interaction with objects in these visualisations on the acquisition of concepts and operations of first-order predicate logic, is studied.

Visualisations

Several studies have tried to explain what gives visual representations so much power. For instance, Paivio (1986) and Clark and Paivio (1991) explained with their theory of dual coding that memory for pictures is better than memory for texts. According to this theory, there are two independent coding systems, the verbal coding system and the image coding system. Information coded in both systems is assumed to be remembered better than information coded in only one system. As texts are processed and encoded in the verbal system and pictures are processed both in the image and in the verbal system, pictures are more likely to be remembered better than text. Furthermore, theories of cognition state that advantages of graphical representations can be explained by the fact that these representations are more effective in communicating material, if they are understood. For instance, Larkin and Simon (1987) described the idea of 'locality': locating semantically related information together in a diagram can support extremely useful and efficient computational processes. Kulpa (1995) and Koedinger and Anderson (1994) described the idea of 'emergence': by coding information in diagrams, new information becomes visible. Stenning

and Oberlander (1995) described the idea of 'inexpressiveness' or 'specificity': diagrams enforce representation of information, leading to weak expressiveness and limited abstraction and thereby facilitating processibility.

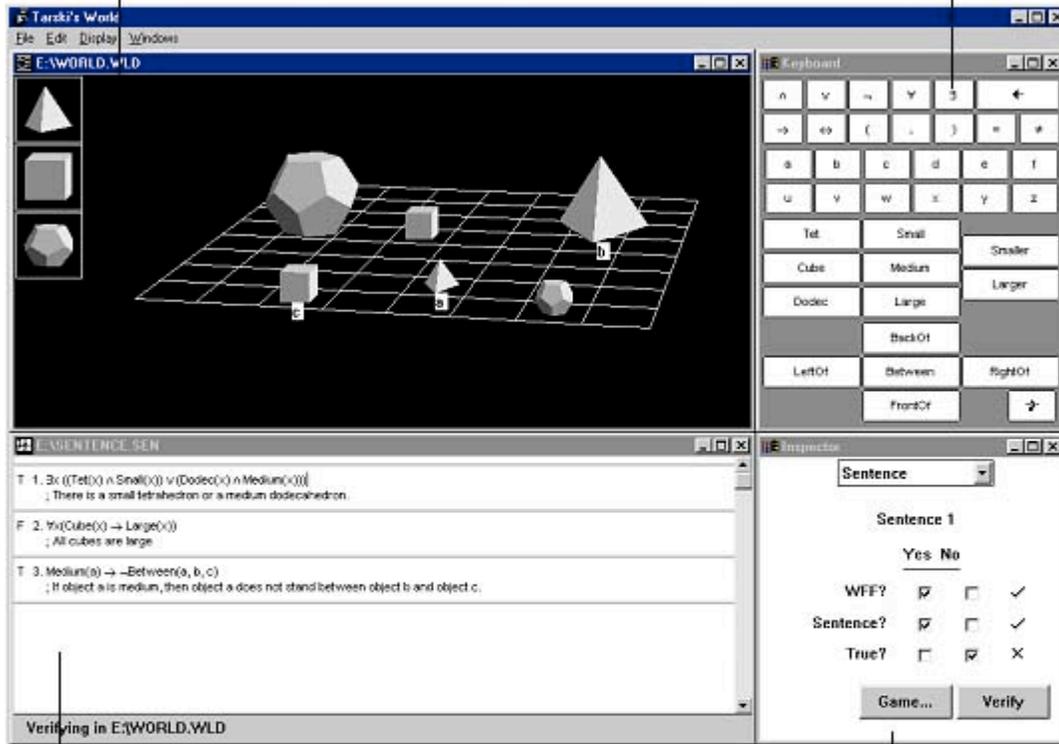
Dependent on the instructional purposes, visual representations can have several functions. For instance, Dijkstra, Jonassen and Sembill (2001) used the following categorisation: (a) the representation can depict the reality (e.g., a photograph, a drawing, a map); (b) the representation can depict a change of state of an object in the reality or an aspect of the reality (e.g., a graph showing the course of temperature in a city over a year); or (c) the representation can show a process occurring in reality (e.g., a demonstration, a simulation). In first-order predicate logic, the function of visual representations is the depiction of the reality. By using visual representations (which then make up the reality rather than the depiction of the reality), the notion of truth is introduced. Logical expressions can be related to the reality and the truth or falsity of the statements in this reality can be determined, so that the meaning of the operators, rules and concepts of logic can be learnt. Furthermore, a visual representation gives the possibility to visually check the reasoning process in the available visualisation and to easily retain the steps made in this reasoning process.

The way in which the reality can be represented in a visualisation can range from a direct, everyday reality (e.g., pictures of everyday life situations) via an entirely pre-structured and well-defined reality (e.g., worlds of geometrical figures) to complete abstraction (e.g., sets of abstract mathematical objects and elements). Both extremities of these levels of abstraction have advantages and disadvantages. The drawback of learning in an everyday life context is that students are tempted to pay attention to irrelevant aspects of the problem, which can easily lead to misunderstandings. When solving problems in an everyday life context, learners will use ideas developed by experiences in everyday life in which pragmatic or user conditional aspects as preferences and hidden assumptions can play a role. Students will use their everyday life ideas (including user conditional aspects) about what is correct reasoning, whereas they should learn to abstract from the given context and learn to reason according to the rules of logic (i.e., the truth conditional aspects). Truth conditional aspects are assumed to be learnt best without any interference of user conditional aspects. Giving reality as a complete abstraction also shows some drawbacks. Learners then receive abstract, conceptual knowledge which is isolated from the situations in which this knowledge is normally used. Learners will not always understand what the concepts and rules are about and the knowledge will not be imbedded into prior knowledge. Therefore, an entirely pre-structured and well-defined world is assumed to be a good choice for representing reality. Such a world makes all operations possible and at the same time shows what happens when certain operations are applied. The world is well defined, in such a way that errors can be precluded, since irrelevant characteristics of the problem situation are left out of the context. Abstract principles are related to concrete objects, so that meaningfulness and understanding can be reached and because the context is controlled, unwanted characteristics of the context will have no influence and learners can focus on the truth conditional aspects without any interference of user conditional aspects. An example of such a pre-structured and well-defined reality is Tarski's World (Barwise & Etchemendy, 1992). Tarski's World is a computer-based learning environment in the domain of first-order predicate logic in which a world consisting of geometrical objects and relations between these objects is used (see Figure 1, world module).

Eysink, Dijkstra and Kuper (2001) conducted a research with first-year social science students in which they investigated the claim that visualisations of a pre-structured and well-defined reality indeed support the development of knowledge and skills in first-order predicate logic. They compared a version of Tarski's World in which the world window consisted of a reality depicted by visual representations to a version of Tarski's World in which the world window consisted of a reality described by language representations. Results showed that there were no differences in learning effects, but that most subjects presented with the language representations produced visualisations themselves, especially when the problem situation increased in complexity. Apparently, the subjects needed the visualisations in order to solve the problems. It is our view, though, that it is better to present the subjects with visualisations than to let them construct the visualisations themselves. This claim is based on the phenomenon which Johnson-Laird (1989) called the 'mental models of discourse'. When language representations are presented, imaginations or mental models arise automatically on the basis of these representations. The mental models make explicit the structure of the situation as it automatically arises from the language representation instead of making explicit the structure of the situation as described exactly by the sentence. As logic is precise, these slightly different imagined visualisations can cause a greater chance of making errors. Therefore, learners should be given visualisations rather than letting them construct these themselves on the basis of language descriptions.

World module: the student can construct a world of geometric objects by mouse clicking on the objects on the left of the screen. The objects can vary in shape (Tet, Cube, Dodec), size (Small, Medium, Large) and position (LeftOf, RightOf, BackOf, FrontOf, Between).

Keyboard module: the student can construct logical formulae by mouse clicking on the keys.



Sentence module: the sentences the student is constructing by using the keyboard module appear in this screen. The computer ignores the semicolons followed by sentences in English. By giving T(ue)s and F(alse)s, the computer gives feedback about the truth value of the sentences in the world given in the world module.

Inspector module: the student can check whether a logical sentence is syntactically correct (WFF?), whether all variables are bound to a quantifier (Sentence?) and whether the formula is true (True?) in the world given in the world module, by selecting a box. The computer gives immediate feedback (✓, ✗).

Figure 1. Tarski' World: world module, sentence module, inspector module and keyboard module.

Interactive visualisations

Piaget (1973) stated that learners need to act in the environment if knowledge development is to ensue. Knowledge is constructed through actions on objects in the environment. He added to this that formal concepts are constructed in a different way from empirical concepts. Although both types are developed by experiencing reality, Piaget distinguished two kinds of experiences in that respect: (1) the physical experience which resembles learning in the experimental sciences, and (2) the logic-mathematical experience which resembles learning in the formal sciences. The physical experience consists of abstracting information from the object itself. For instance, a child picking up balls of different sizes experiences different weights and can infer certain general rules from this. The logic-mathematical experience, however, consists of abstracting knowledge by *operating* on the objects and not of abstracting knowledge from the objects themselves. In addition to characteristics already present, new characteristics are attributed to objects. Experience, then, refers to the relation between the characteristics attributed to the objects by operating on or interacting with them, and not to the characteristics the objects already possess. In this sense, knowledge is seen to be abstracted from the operations as such and not from the physical features of the object. For instance, a child learns the concept of order by ordering different balls to size. In this case, size is a physical feature all balls possess, order is a logic-mathematical concept being added by operating on the balls. At a certain moment, the applications of operations on physical objects become superfluous and the logic-mathematical operations are being integrated in symbolic operators, which can be applied in different contexts. Therefore, from a certain moment, pure logic and mathematics are left, for which no (concrete) experience is needed. Formal concepts and operations can be abstracted from reality and these representations can be operated on mentally. It is our view, however, that learning should gradually shift from physical operations on concrete objects to mental operations on abstract objects. What is first developed by (physical and logic-mathematical) experience in situations with concrete objects, can (at a later stage) be logically deduced, too. An interactive visual representation of geometrical objects is thus assumed to support learning logic. Besides a visual representation of geometrical objects, Tarski's World (Barwise & Etchemendy, 1992) also meets the requirement of giving the learners the possibility to interact with the objects in the visualisation. The programme consists of four main components (see Figure 1): (1) the world module or interactive visualisation, in which the students can place the objects of a certain size and shape in the proper position; (2) the sentence module, in which the formal sentences appear; (3) the keyboard module for constructing sentences in the sentence module; and (4) the inspector module, in which sentences from the sentence module can be checked to verify whether they are well-formed, syntactically correct, and true/false in relation to the world in the world module. In a typical problem, students are given natural language sentences which they have to translate into logical expressions. When all sentences are translated, a world can be constructed. In the world module, the students can click on the objects on the left side of the screen. The chosen object then appears on the grid and can be given another position, size, or shape, and can be given a name. After constructing a world, the students can start to evaluate their actions in the inspector module. In the inspector module, three questions can be asked. Two concern the translation of the natural language sentences into the logical expressions ("WFF?" and "Sentence?") and one concerns the relation between the sentences and the world ("True?"). After evaluating, the logical expressions and/or the world can be changed and new feedback can be asked. By changing the world, the truth values of the sentences change. By systematically interacting with the objects in the visualisation, the learners can experience what actions have which effect. The truth conditional aspects can be induced and knowledge about the meaning of concepts and operations can be developed.

In order to gain empirical support for the claim that interaction with objects supports the development of knowledge and skills in first-order predicate logic, Eysink, Dijkstra and Kuper (2001) designed an experiment with Tarski's World in which the effect of interactive visual representations was compared to the effect of static visual representations. The results of this experiment showed that subjects who were given the possibility to interact in the visualisation did not outperform subjects not given this possibility. However, a closer look and further analysis of the data showed that this result could be explained by the fact that subjects did not fully use the possibilities of the interactive visualisation. They played around in such a way that they were confronted with the subject matter they already understood, but they did not confront themselves with the subject matter put in a new situation. The subjects were tempted to stay in familiar situations, even when given freedom to explore. They repeatedly checked things they already knew, instead of trying out something new and they headed straight for the solution without straying from their path, even if this could have resulted in a better solution. Apparently, most learners were not able to handle the freedom they got when confronted with interactive visualisations. Therefore, Eysink, Dijkstra and Kuper (in press) designed a new experiment in which they tried to stimulate the learners to interact with the visualisations by giving them problems in which they were instructed to do so. In this experiment, the effect of interactive visualisations became visible. Differences were found between learners who had the possibility to interact with the

objects in the visualisation and learners who did not have this possibility. However, this difference appeared only in the long run. There were no immediate effects, but after two weeks, the learners who were stimulated to interact in the visualisation remembered more of the subject matter than learners who could not interact with the objects. Furthermore, again no differences in learning effects were found between subjects who had the possibility to interact with the objects in the visualisation but who were not instructed to use this possibility and subjects who did not have the possibility of interaction at all. The results thus showed that interactive visualisations are a helpful component to include in instruction in order to facilitate learning logic, but only in combination with instructions in which the learners are stimulated to interact with the objects in the visualisation.

Discussion

The experiments summarised in this paper tried to answer the research question whether instructional variables originating from empirical sciences, such as visualisations and interaction with objects in these visualisations, can also be used to develop knowledge and skills in first-order predicate logic. From the results of the experiments, it appears that this question can be answered positively. The experiments showed that learners who were presented with verbal descriptions of the problem situation apparently needed to construct additional visual representations to reach the same results as learners who were given these visualisations. Questions can arise, however, whether the type of visualisation influenced the results. It was our view that the visualisation should consist of a completely pre-structured and well-defined world, because only then differences in truth and user conditional aspects that play a role in everyday life can be controlled, so that the chance of interference with the learning process is small or even absent. This claim was not studied empirically, though, so that it is possible that other types of visualisations such as worlds consisting of everyday life objects might have given the same results. Furthermore, the visualisations used in the experiments consisted of objects having spatial relations to each other (e.g., to the left of, behind, between). It is possible that these spatial relations automatically evoked mental images, so that learners could not reason without these visualisations, whereas other visualisations would not have automatically evoked mental images. This could account for the positive effects of visualisations consisting of objects which have spatial relationships to each other.

The advantage of interacting with objects in a visualisation was shown in the last experiment. Learners who had the possibility to interact with objects and who were stimulated to do so remembered the subject matter better after two weeks compared to learners who did not have the possibility to interact. By relating the logical statements to an interactive visual representation, learners can develop knowledge and skills from logic-mathematical experiences in this visualisation by interacting with objects and abstracting knowledge about logical concepts and operators from their operations. This instructional variable is regarded as useful to teach logic. The main lesson that can be learnt here, though, is that giving the learners freedom to explore in interactive visualisations is only effective when this possibility to interact is combined with the stimulation to interact, as learners do not interact on their own accord. Therefore, it is very important to give some extra help, which can be done in various ways. We chose for problems in which the learners were instructed to interact with the visualisation. Other ways of regulating the problem solving process of the learners, such as registering all experiments which the learner performed (de Jong & van Joolingen, 1998), can possibly lead to facilitating effects as well. In Tarski's World, this would mean registering all checks of combinations of sentences and worlds and their truth values, so that the learners have an overview of what they have been doing.

Furthermore, the effect that learners do not fully use the possibilities of interactive visualisations can serve as a starting point for a more general discussion on interactive learning environments. Questions arise, such as, how 'open' should interactive learning environments be? To what extent can learners be left on their own? To what extent do they need at least some kind of instructional support or guidance in their interactions? Can learners be held responsible at all for their own development of knowledge and skills? Is the extent of required guidance age-bound? What influence do these open learning environments have on the quality and quantity of the subject matter learnt? And what about the effects in the long run? All these (and other) questions are important for the design of interactive learning environments and for the design of instruction. The results of the present study can be used as a starting point in an attempt to answer some of them.

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